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ÉQUATIONS AUX DÉRIVÉES PARTIELLES

ON REFLECTION OF SHOCK FRONT IN MULTIDIMENSIONAL SPACE

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1 Introduction

When a shock wave produced by an explosion or by a fast flying projectile hits an obstacle, the reflection of shock waves occurs. To determine the place of the reflected shock front and the flow field behind the shock is the main subject, which we are going to study.

If the shock front and the surface of the obstacle are planes parallel to each other, and the shock moves towards the obstacle with constant velocity, then the reflection problem has been solved by Courant and Friedrichs in [1]. Later, in the two dimensional case (steady flow in two dimensional case or unsteady flow in one space dimensional case), this problem was solved locally by Gu Chaohao and others (see [2],[3]). However, in the case with more independent variables, such problem is still open. Recently, by means of microlocal analysis A. Majda [4,5] and G. Métivier [6] studied problems on local existence of shock front solution and interaction of two shocks for the system of conservation laws. On the basis of these works we are able to deal with the problem on reflection of shock waves.

Our main purpose in this paper is to prove the local existence of the solution for the problem on reflection of shock waves in multi-dimensional case. First, we formulate the original problem as a Goursat problem for the system of conservation laws with one free boundary and another fixed characteristic boundary. Then after reducing it to a nonlinear Goursat problem with two fixed boundary we find an asymptotic solution. Finally, by Newton's iteration we establish a convergent sequence, and the limit offers a solution for the nonlinear problem. At each step including determination of the first term in the sequence we need to solve a linearized Goursat problem and derive corresponding estimates. The details are referred to [10].

2 Formulation

Let us consider the reflection of shock front for inviscid unsteady flow. For the notational simplicity we only consider the case of space-dimension 2. The system of conservation laws is

$$(2.1)\qquad \frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho(e+\frac{1}{2}q^2)\end{pmatrix}+\frac{\partial}{\partial x}\begin{pmatrix}\rho u\\ p+\rho u^2\\ \rho uv\\ \rho u(i+\frac{1}{2}q^2)\end{pmatrix}+\frac{\partial}{\partial y}\begin{pmatrix}\rho v\\ \rho uv\\ p+\rho v^2\\ \rho v(i+\frac{1}{2}q^2)\end{pmatrix}=0$$

where (u, v) are the components of velocity, $q^2 = u^2 + v^2$, p, ρ, e, i represent pressure, density, inner energy, enthalpy respectively. On the shock front the following Rankine-Hugoniot conditions must be satisfied :

$$(2.2)\qquad \begin{bmatrix}\rho\\ \rho u\\ \rho v\\ \rho(e+\frac{1}{2}q^2)\end{bmatrix}\psi_t-\begin{bmatrix}\rho u\\ p+\rho u^2\\ \rho uv\\ \rho u(i+\frac{1}{2}q^2)\end{bmatrix}+\begin{bmatrix}\rho v\\ \rho uv\\ p+\rho v^2\\ \rho v(i+\frac{1}{2}q^2)\end{bmatrix}\psi_y=0$$

where $x = \psi(t, y)$ is the equation of shock front and [ ] represents the jump of corresponding quantity across the shock front. For the rigid wall as a fixed boundary, the boundary condition is

$$n_x u + n_y v = 0. \tag{2.3}$$

where (n_x, n_y) is the normal direction of the boundary. Let Σ be a given surface of obstacle, S be an incident shock front. Suppose that the flow ahead of S is static and we know the motion of the shock wave in the case when the obstacle is absent (e.g. an explosive shock wave moves into static flow). At $t = 0$, S meets Σ at point 0, then for $t > 0$ the reflection of shock wave occurs. Now we are going to determine the reflected shock front $t > 0$ and the flow field behind the reflected shock front.

We choose the coordinates such that point 0 is the origin, the equation of Σ is $x = \phi(y)$, where $\varphi(y)$ is of C^∞ and $\varphi(0) = \varphi'(0) = 0$, $\varphi(y) \leq 0$. The equation of S is $x = \chi(t, y)$ on the left side and the right side the parameters of flow field are u_a, v_a, p_a, ρ_a and $u_b, v_b, p_b \rho_b$ respectively. Since the incident shock front for Euler system is an extreme shock front, then the appearence of Σ will not influence the motion of S and change the flow field until it meets S. The intersection $\Sigma \cap S$ can be determined by the equation $x = \varphi(y)$ and $x = \chi(t, y)$, we denote it as

$$\sigma \begin{cases} x = \varphi(y) \\ t = \theta(y) \end{cases} \tag{2.4}$$

Denoting the equation of the reflected shock front S_1 issued from σ by $x = \psi(t, y)$, the expected solution of (2.1) can be written as

$$U(t, x, y) = \begin{cases} U_c(t, x, y) & , \quad \varphi(y) < x < \psi(t, y) \\ U_0(t, x, y) & , \quad x > \psi(t, y) \end{cases} \tag{2.5}$$

where U is the abbreviation of (u, v, p, ρ). Our result in the paper is :

Theorem.— *Suppose that the problem on reflection of shock front is formulated as above,* $\varphi(y), \chi(t, y), U_a(t, x, y), U_b(t, x, y)$ *are* C^∞ *smooth, then there exist* C^∞ *function* $\psi(t, y)$ *and* $U_c(t, x, y)$ *near the origin, such that* $U_c(t, x, y)$ *satisfies (2.1) in* $\phi(y) < x < \psi(t, y)$*, (2.2) on* $x = \psi(t, y)$ *and (2.3) on* $x = \varphi(y)$*.*

3 Orientation.

We view the reflection problem as a nonlinear Goursat problem with one characteristic boundary and another non-characteristic free boundary. On the edge σ, by freezing our problem at a point we can find the parameters of the flow field on σ behind the reflected shock front and the slope of the shock front at each point on σ. In this stage we use the known results on reflection of plane shock wave with constant velocity attacking a plane wall, the relative formula and computation can be found in [1].

By a coordinate transformation

$$t' = t - \theta(y)\ ,\ x' = x\ ,\ y' = y. \tag{3.1}$$

the edge σ will be placed on $t' = 0$. Then by another coordinate transformation

$$\begin{aligned} x_1 &= t\frac{x-\varphi(y)}{\psi(t,y)-\varphi(y)} \\ x_2 &= t\frac{\psi(t,y)-x}{\psi(t,y)-\varphi(y)} \\ y &= y \end{aligned} \tag{3.2}$$

the problem is reduced to a new nonlinear Goursat problem with two fixed boundary. The image of Σ is $x_1 = 0$ which is still a characteristic boundary, and the image of S is $x_2 = 0$, which is non characteristic. Here we notice

$$\frac{\partial(x_1,x_2,y)}{\partial(t,x,y)} = -t(\psi-\varphi)^{-1} \neq 0$$

it means the transformation (3.2) is non singular.

The nonlinear problem now has the form

$$\begin{aligned} &A(U,\psi)\frac{\partial U}{\partial x_1} + B(U,\psi)\frac{\partial U}{\partial x_2} + Q(U,\psi)\frac{\partial U}{\partial y} = 0, \quad x_1 > 0, x_2 > 0\ ; \\ &\ell\, U = 0,\ x_1 = 0\ ; \\ &F(x_1,y,U,\psi,\nabla\psi) = 0\ , \quad x_2 = 0\ ;\ \psi(0,y) = 0. \end{aligned} \tag{3.3}$$

The linearization of (3.3) is

$$\begin{aligned} &A\ \frac{\partial\ \delta U}{\partial x_1} + B\ \frac{\partial\ \delta U}{\partial x_2} + Q\ \frac{\partial\ \delta U}{\partial y} = f \quad ,\ x_1 > 0\ ,\ x_2 > 0\ ; \\ &\ell\ \delta U = 0 \quad , \quad x_1 = 0\ ; \\ &p\frac{\partial\delta\psi}{\partial t} + q\frac{\partial\delta\psi}{\partial y} + h\delta\psi + m\delta U = g \quad , \quad x_2 = 0\ ;\ \delta\psi(0,y) = 0. \end{aligned} \tag{3.4}$$

The boundary $x_1 = 0$ is characteristic and it can be embedded in a family of characteristic surfaces. The boundary $x_2 = 0$ is noncharacteristic one, satisfying uniform Lopatinski conditions (see [4]).

Next our main steps are :

1) Obtain the existence of solution for the linearized problem (3.4) and establish corresponding energy estimates.
2) Find same asymptotic solutions of nonlinear problem (3.3)
3) Using Newton's iteration scheme to construct a convergent sequence of approximate solutions, the limit of the sequence is the solution of (3.3).

4 Discussion on the linearized problem

The existence of solution for the linearized problem is valid according to [7-9]. In order to improve the smoothness and establish the energy estimates we introduce some notations as follows :

$$x = (x_1, x_2, y)$$

$$\Omega_T = \{X / x_1 > \theta; x_2 > 0\ ;\ x_1 + x_2 < T\}$$

$$\omega_T = \{(t, y)/0 < t < T\}$$

$$V = (x_1\partial_{x_1}, \partial_{x_2}, \partial_y) \quad ; \quad D = \partial_{x_1}$$

$$\alpha = (\alpha_{x_1}, \alpha_{x_2}, \alpha_y) \quad ; \quad \beta = (\beta_{x_1}, \beta_{x_2}, \beta_y).$$

$$\gamma = (\gamma_t, \gamma_y)$$

$$L^2_\lambda(\Omega_T) = \{u/(x_1 + x_2)^{-\lambda} u \in L^2(\Omega_T)\}$$

$$L^2_\lambda(\omega_T) = \{f/t^{-\lambda} f \in L^2(\omega_T)\}$$

$$H^{r,k}_\lambda(\Omega_T) = \{u/V^\beta D^s u \in L^2_{\lambda-\beta_{x_2}-s}(\Omega_T),\ s + |\beta| \le r + k\ ,\ s \le r\}$$

$$B^k_\lambda(\Omega_T) = \bigcap_{r \le \frac{k}{2}} H^{r,k-2r}_\lambda(\Omega_T)$$

$$H^k_\lambda(\omega_T) = \{f/\partial^\gamma f \in L^2_{\lambda-\gamma_t}(\omega_T), |\gamma| \le k\}$$

$$\|U\|_{H^{r,k}_\lambda(\Omega_T)} = \left\{ \sum_{\substack{s+|\beta| \le k+r \\ s \le r}} \lambda^{2(r+k-s-|\beta|)} \|V^\beta D^s U\|_{L^2_{\lambda-\beta_{x_2}-s}} \right\}^{1/2}$$

$$\|U\|_{B^k_\lambda(\Omega_T)} = \left(\sum_{r \le k/2} \|U\|^2_{H^{r,k-2r}_\lambda(\Omega_T)} \right)^{1/2}$$

$$\|f\|_{H^k_\lambda(\omega_T)} = \left(\sum_{|\gamma| \le k} \lambda^{2(k-|\gamma|)} \|\partial^\gamma f\|^2_{L^2_{\lambda-\gamma_t}} \right)^{1/2}$$

Next we are concentrating on establishing the energy estimates. The expected estimate is
(4.1)

$$\lambda\|\delta U\|^2_{B^k_{\lambda+\frac{1}{2}}(\Omega_T)} + \|\delta U\|^2_{H^k_\lambda(\omega_T)} + \|\delta\psi\|^2_{H^{k+1}_{\lambda+1}(\omega_T)} \le C\left(\frac{1}{\lambda}\|f\|^2_{B^k_{\lambda-\frac{1}{2}}(\Omega_T)} + \|g\|^2_{H^k_\lambda(\omega_T)}\right).$$

To establish the desired estimate we use :

a) Transformation of independent variables and unknown functions :

$$(4.2) \qquad j\begin{cases} t = x_1 + x_2, \\ \theta = \dfrac{x_1}{x_1 + x_2}, \\ y = y\ ; \end{cases}$$

$$J_\lambda \begin{cases} u(x_1, x_2, y) \mapsto J_\lambda u(t,\theta,y) = t^{-\lambda} u(t\theta, t(1-\theta), y), \\ \varphi(t,y) \mapsto J_\lambda\varphi(t,y) = t^{-\lambda}\varphi(t,y). \end{cases}$$

b) Dyadic partition of unity :

$$(4.4) \qquad \begin{aligned} u(t,\theta,y) &= \sum u_j(t,\theta,y) = \sum \chi(2^j t) u(t,\theta,y), \\ \varphi(t,y) &= \sum \varphi_j(t,y) = \sum \chi(2^j t)\varphi(t,y), \end{aligned}$$

where $\chi \in C_0^\infty(\mathbf{R}), \operatorname{supp}\chi \subset (\frac{1}{2}, 2)$ and $\Sigma\chi(2^{-j}t) = 1$.

c) Dilation :

$$(4.5) \qquad \begin{aligned} \tilde{u}_j(t,\theta,y) &= 2^{-\frac{j}{2}} u_j(2^{-j}t, \theta, y), \\ \tilde{\varphi}_j(t,y) &= 2^{-\frac{j}{2}} \varphi_j(2^{-j}t, y). \end{aligned}$$

After these procedure the original estimate is reduced to an estimate of regular Sobolev norm (without special weight $(x_1 + x_2)^{-1}$ or $t^{-\lambda}$) of solution for an initial boundary value problem in the domain $\{0 < \theta < 1, \frac{1}{2} < t < 1\}$ with normal size. Since the domain is away from $t = 0$, we do not need worry about degeneracy of the system on $t = 0$ any more. Meanwhile, we notice that the boundary $\theta = 0$ is characteristic, and the boundary $\theta = 1$ is noncharacteristic, on the later the uniform Lopatinski condition is satisfied. By using another partition of unity with respect to $\theta \in (0,1)$, we can separate the estimate near the boundary $\theta = 0$ and $\theta = 1$. Near $\theta = 1$ we use Majda's result in [4], and near $\theta = 0$ we use the theory of symmetric hyperbolic system.

Let us give more explanation on the estimate near $\theta = 0$. Near this boundary the estimate (4.1) after transformation a) - c) becomes

$$(4.6) \qquad \lambda\|u\|^2_{B^k_{(\lambda)}} \le \frac{C}{\lambda}\|L'u\|_{B^k_{(\lambda)}}.$$

where u is supported in $\frac{1}{2} < t < 1$, $0 < \theta < \theta_0$ with $\theta_0 < 1$, L' is a slight modification of L, it is still a symmetric hyperbolic operator, and

$$\|u\|_{B^k_{(\lambda)}} = \left(\sum_{r \le \frac{k}{2}} \sum_{\substack{|\beta| \le k-2r \\ b \le r}} \lambda^{2(k+r-b-|\beta|)} \|V^\beta D^b U\|^2_{L^2} \right)^{1/2} .$$

By a standard procedure as acting tangential operators on the both sides of the system and integrating by parts, it is easy to have an estimate for $\|u\|_{H^{0,k}_{(\lambda)}}$. Next we are going to show that starting from the estimate for $\|u\|_{H^{0,k}_{(\lambda)}}$ we can obtain the estimate for $\|u\|_{B^k_{(\lambda)}}$. Without loss of generality we take $\lambda = 1$ and write the system by a coordinate transformation as

$$A_0 \frac{\partial u}{\partial t} + A_1 \frac{\partial u}{\partial x} + A_2 \frac{\partial u}{\partial y} + Bu = f, \tag{4.7}$$

where $A_0 > 0$, $A_1 = \begin{pmatrix} R & 0 \\ 0 & 0 \end{pmatrix}$ with R being a nonsingular matrix. Splitting u as (u_a, u_b) corresponding to the block form of A_1 we have

$$\frac{\partial u_a}{\partial x} = R^{-1} \left(f_a - (A_0)_a \frac{\partial u}{\partial t} - (A_2)_a \frac{\partial u}{\partial y} - B_a u \right), \tag{4.8}$$

$$(A_0)_{bb} \frac{\partial u_b}{\partial t} + (A_2)_{bb} \frac{\partial u_b}{\partial y} + B_{bb} u_b = f_b - (A_0)_{ba} \frac{\partial u_a}{\partial t} - (A_2)_{ba} \frac{\partial u_a}{\partial y} - B_{ba} u_a, \tag{4.9}$$

where the subscripts a, b indicate the corresponding blocks. Differentiating (4.9) with respect to x implies

$$(A_0)_{bb} \frac{\partial (u_b)_x}{\partial t} + (A_2)_{bb} \frac{\partial (U_b)_x}{\partial y} + B_{bb} (u_b)_x = F_b. \tag{4.10}$$

Now since we have obtained an estimate for $\|u\|_{H^{0,k}}$, from (4.8) we know $\|u_a\|_{H^{1,k-1}}$, then from (4.10) we can obtain the estimate for $\|(u_b)_x\|_{H^{0,k-2}}$, and $\|u_b\|_{H^{1,k-2}}$ as well. Using (4.8) and (4.10) alternatively the estimate for $\|u\|_{B^k}$ can be obtained.

We mention that in the linearized system (3.4), the coefficients depend on U, ψ. Therefore, even for obtaining the estimate $\|\delta U\|_{H^{0,k}}$, we still need to check how much derivatives of U and ψ are involved. In order to sketch that let us denote δU by u and assume that all coefficients depend only on U, because the dependency of all coefficients on ψ does not cause more trouble. Usually, when we estimate $\|V_{\sigma_1} \dots V_{\sigma_k} u\|_{L^2}$ with V_{σ_j} being tangential differential operator, we need to control the term $\| [V_{\sigma_1} \dots V_{\sigma_k}, L] u\|_{L^2}$. However, it is possible that in the expression $[V_{\sigma_1} \dots V_{\sigma_k}, L] u$ there will appear non-tangential derivatives of u now. So we neeed to use the following facts to get rid of the difficulty.

1. We split u to (u_a, u_b) as above, the nontangential derivatives of u_a can be expressed by tangential ones of u.
2. If M is a tangential differential operator with coefficients depending on $U \in B^k$, then we have

$$[V_{\sigma_1} \dots V_{\sigma_s}, M] = \sum_{r \leq s} \sum_{\tau_1 \dots \tau_r} C_{\sigma_1 \dots \sigma_s \tau_1 \dots \tau_r} \tag{4.11}$$

where $C_{\sigma_1 \dots \sigma_s \tau_1 \dots \tau_r}$ can be expressed as :

q-form : $q^{\tau_1 \cdots \tau_r}_{\sigma_1 \dots \sigma_s} V_{\tau_1} \dots V_{\tau_{r-1}} D_r$ with $q^{\tau_1 \cdots \tau_r}_{\sigma_1 \dots \sigma_s}$ depending on U and tangential derivatives up to $(s - r + 1)^{-th}$ order.

or

p-form : $p^{\tau_1 \cdots \tau_r}_{\sigma_1 \dots \sigma_s} V_1 \dots V_{\tau_r}$ with $p^{\tau_1 \cdots \tau_r}_{\sigma_1 \dots \sigma_s}$ depending on v and its derivatives up to $(s - r + 2)^{-th}$ order, including at most once in non tangential direction.

Alternatively using q-form or p-form of the expression $C_{\sigma_1 \dots \sigma_s, \tau_1 \dots \tau_r}$ for the case $r \leq \frac{k}{2}$ or $r > \frac{k}{2}$, we can obtain $\| [V_{\sigma_1} \dots V_{\sigma_s}, M] \, u \| \leq C(\|U\|_{B^k}).\|u\|_{B^k}$ which leads $\| [V_{\sigma_1} \dots V_{\sigma_s}, L] \, u \| \leq C(\|V\|_{B^k}).\|u\|_{B^k}$. Remember that $\|\delta U\|_{B^k}$ can be controlled by $\|\delta U\|_{H^{0,k}}$, we are successful in estimating $\|\delta U\|_{H^{0,k}}$, and then $\|\delta U\|_{B^k}$.

5 Treatment on nonlinearity.

As we showed in paragraph 3, first we look for asymptotic solutions for the nonlinear problem (3.3). The first asymptotic solution is $U_0 = V(y)$, $\psi_0 = \tau(y)t$, where $V(y), \tau(y)$ satisfy

$$\begin{aligned} \ell \, V(y) &= 0 \\ F(0, y, V(y), 0, \tau(y)) &= 0. \end{aligned} \tag{5.1}$$

In fact, $V(y), \tau(y)t$ are the solution for the frozen problem at each point on σ. Define U_j, ψ_j successively by

$$\begin{aligned} & L_0 V_j = -L(U_j, \psi_j) U_j \\ & \ell \, \gamma_1 V_j = 0 \\ & \begin{cases} F_0(V_j, \chi_j) = -\mathcal{F}(U_j, \psi_j), \\ \chi_j|_{t=0} = 0 \end{cases} \end{aligned} \tag{5.2}$$

$$\begin{aligned} U_{j+1} &= U_j + V_j, \\ \psi_{j+1} &= \chi_j + \psi_j. \end{aligned}$$

where

$$L_0 V_j = A(U_0, \psi_0) \frac{\partial V_j}{\partial x_1} + B(U_0, \psi_0) \frac{\partial V_j}{\partial x_2},$$

$$F_0(V_j, \chi_j) = p(U_0, \psi_0) \frac{\partial \chi_j}{\partial t} + h(U_0, \psi_0) \chi_j + m(U_0, \psi_0) V_j,$$

we can verify

$$
(5.3) \qquad \begin{aligned} L(U_j, \psi_j)U_j &= 0((x_1+x_2)^j) \\ \mathcal{F}(U_j, \psi_j) &= 0(t^{j+1}) \\ \ell\, U_j &= 0. \end{aligned}
$$

For large j the right side of (5.3) has bounded Sobolev norm with weight $(x_1+x_2)^{-1}$ or $t^{-\lambda-1}$. Fix j, set $(U^{(0)}, \psi^{(0)}) = (U_j, \psi_j)$. By Newton's iteration scheme we construct a sequence $\{U^n, \psi^{(n)}\}$ as

$$
(5.4) \qquad \begin{cases} L(U^{(n)}, \psi^{(n)})W^{(n+1)} = -L(U^{(n)}, \psi^{(n)})U^{(0)}, \\ \ell\, W^{(n+1)} = 0, \\ \begin{cases} F_{U^{(n)},\psi^{(n)}}(W^{(n+1)}, \theta^{(n+1)}) = -\mathcal{F}(U^{(n)}, \psi^{(n)}) + F_{U^{(n)},\psi^{(n)}}(W^{(n)}, \theta^{(n)}); \\ \theta^{(n+1)}|_{t=0} = 0, \end{cases} \\ U^{(n+1)} = U^{(0)} + W^{(n+1)}, \\ \psi^{(n+1)} = \psi^{(0)} + \theta^{(n+1)}. \end{cases}
$$

Regarding the process from $(U^{(n)}, \psi^{(n)})$ to $(U^{(n+1)}, \psi^{(n+1)})$ as a map π, we can prove the map is an inner map in the set

(5.5)

$$
W^k_{\lambda,T} = \left\{(U,\psi), U \in B^k_{\lambda+\frac{1}{2}}(\Omega_T), \ell_1 U|_{x_1=0} = 0\ ,\ U|_{x_2=0} \in H^k_\lambda(\omega_T), \psi \in H^{k+1}_{\lambda+1}(\omega_T)\right\},
$$

if T is small, λ is large and $k \geq 10$. Moreover, π is contractive for a weaker norm. Thus the sequence $\{(U^{(n)}, \psi^{(n)})\}$ is convergent, and the limit offers a solution for the nonlinear problem.

6 Remarks.

Remark 1.

In paragraph 2 we assume that the motion of the shock wave is known, if the obstacle does not appear. This assumption can be relaxed. We appreciate that G. Métivier gives a good comment on that.

Remark 2.

Our discussion is also available for reflection of shock front for inviscid steady flow in three dimensional case. The conclusion is : if the frozen problem at some point on the intersection of the shock and the obstacle has a solution, then the reflection problem in three dimensional space can also be solved locally near the given point.

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